INTRODUCTION TO MULTILEVEL MODELS

CSDA Fall Workshop
Jeffrey Napierala
CSDA & Department of Sociology
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Outline

■ Introductory Information
  - What is a MLM? Do’s and Don’ts...
  - Key Concepts
  - Theory
  - Data
  - Software
■ Take a Break?

Outline (part 2)

■ SAS Tutorial
  - Code conversions on UCLA website: http://www.ats.ucla.edu/stat/ProcModeling/Models/
■ Example: Two level Models
■ Example: Individual Growth Models
■ Conclusion
■ Q & A
What is a MLM?

- Terminology
  - Multilevel Model (MLM) = Hierarchical Linear Model (HLM) = Mixed-Effects Model
  - There is a lot of different terms and symbols used to discuss MLMs. Try to focus on the concepts.
- Designed to examine various forms of “nested” data:
  - The classic example from education is students in classrooms
  - Also, for example, individuals in neighborhoods/organizations
  - observations in time (for individuals), and
  - they can also be data collected using clustered sampling

Why use a MLM?

- They can model inter- and intra-unit variation.
  - Variation “between” and “within” units.
- MLMs can simultaneously test effects at multiple “levels”.
  - Often contextual variables on an individual-level outcome.
- They can test “cross-level” interactions.
  - How individual-level variables vary with contextual variables.
- MLMs can correct for inflated type I error (as exists in models with nested data).
- They provide flexibility for handling many (sometimes complex) types of relationships between variables and within error terms.

When not to use a MLM...

- Very little dependence between observations
  - Results will not change much...
- Very few clusters (also, potentially, a very large number)
  - MLMs may have difficulty converging
  - Might need to try another method
- You just need to control for clustering (non-independence). Other methods are simpler and often provide similar results:
  - Can use cluster-robust standard errors
  - Incorporate design effects (old school)
  - Various survey regression methods
- Your intended audience is unfamiliar with MLM’s and/or complex statistical techniques
Key Concepts

- Level 1 vs. Level 2 (vs. Level 3 ...)
  - Level 1 units are nested within Level 2 units...
- Random vs. Fixed effects
  - Random effects are an additional variance term. They allow the term to vary "randomly" for each group.
  - Fixed effects do not. The coefficient is "fixed" to be the same for each group.
- Variance or deviation from the mean—variance components. There are 3 types:
  - variance in intercepts aka level 2 error
  - variance in slopes aka random effect
  - variance in the level 1 observations aka level 1 error (also sometimes referred to as a random effect)

Key Concepts: How does an MLM Work?

- In theory, fitting a MLM is similar to fitting multiple regression to each level 2 unit.
  - The means and variances from the a MLM would correspond to the mean and variances of the intercepts and slopes of the regressions.
  - In an actual MLM model this is done, essentially, simultaneously.
- OLS vs. ML vs. REML
  - OLS: regression is solved with an equation. There is exactly one set of solutions.
  - ML: "regression" is solved through an iterative process to provide the best fit to the data.
  - REML: Similar to ML, but uses the residuals instead of the observed data. Has better performance in small samples. Usually the default for MLM packages.
  - You should use REML to obtain estimates of the fixed and random effects, but ML to obtain measures of model fit when comparing models.

A Highly Stylized Example: Fixed Slopes and Intercepts
A Highly Stylized Example: Fixed Slopes and Random Intercepts

A Highly Stylized Example: Random Slopes and Fixed Intercepts

A Highly Stylized Example: Random Slopes and Intercepts
Theoretical Considerations

- Be sure your theory fits with your levels.
  - Neighborhood=block or block group or tract or zip codes or census places?
  - City=City or Metro or Metropolitan area (what about consolidated metros)?
  - States? Regions? ...
  - Differences in the unit can result in differences in your findings!
- OMG MAUP!!!! The Modifiable Areal Unit Problem...
- If theories are underdeveloped/non-existent be prepared for findings that are difficult to explain.
- Remember the ecological fallacy: group characteristic ≠ individual characteristic
  - Same holds true for the reverse--the Atomistic Fallacy.

Data Structure

- Cases are nested in larger, aggregate units (e.g. students in classrooms)
- Usually, cases can only belong to one group and are hierarchically organized
  - If not, data are “cross-nested” or “cross-classified,” or have “multiple memberships,” and special models are needed. Can still be run in the MLM framework.
- Dependent variable is at lowest level (e.g. students).
- Independent variables at any level (e.g. students or classrooms or school districts).

Data Structure (organization)

- Long vs. wide for data with repeated measurements
  - Long: each measurement is an observation (row in dataset)
  - Wide: each individual is an observation, with measurements coded into uniquely defined variables (so SES_1, SES_2, SES_3...)
- You want “long” data where values for the level-2 variable are repeated for each observation.
  - Some reorganization help for SAS users:
    - http://www.ats.ucla.edu/stat/sas/modules/wtol_transpose.htm
    - See also the appendix to the Singer article.
Software

- SAS
  - It is arguably the most powerful data management software.
  - It is heavily used in some industries, government agencies, and academic fields.
  - Can conduct a wide variety of MLMs in SAS including those GLMM and MLMs for large data sets.
    - PROC MIXED for normally distributed DVs
    - PROC GLIMMIX for non-normal DVs
    - PROC HPMIXED for large datasets (normal DVs only)
    - PROC NLMIXED for the most complex analyses
  - There is a free version for academics!
  - It is a bit slow, but it does the job: http://www.sas.com/en_us/learn/analytics-
  - There are a wide variety of error-covariance structures available.
    - Tip: the PROC GLIMMIX manual has a more interpretable list than the PROC MIXED manual: http://support.sas.com/documentation/cdl/en/statug/63962/HTML/default/viewer.htm#statug_glimmix_sect009.htm

Software (many options)

- R
  - It is free.
  - Has a number of packages that can run MLMs.

- Others:
  - HLM – commonly used
  - Stata – another good choice
  - Mplus – built around a SEM framework (see: https://www.statmodel.com/download/usersguide/Chapter9.pdf)
  - MLwin
  - S-Plus
  - WinBugs (free)

Some additional thoughts:
- You may need to shop around if you have special analysis needs although both R and SAS have a wide range of options.
- We have a number of these software programs in the BA B-18 computer lab.

SAS TUTORIAL

Two-Level School Effects Models

Files for the workshop at:
http://csda.cas.albany.edu

Data for other packages at:
OLS with one covariate

- Before we get into more complex models...

\[ Y_i = \beta_0 + \beta_1 X_i + \epsilon_i \] where \( \epsilon_i \sim N(0, \sigma^2) \)

- \( i \) indexes students
- \( \beta_0 \) = intercept
- \( \beta_1 \) = slope of the variable \( X_i \)
- \( \epsilon_i \) = residual (error term), which is (assumed to be) normally distributed (and iid) with a mean of 0 and a variance of \( \sigma^2 \) (sigma squared).
- Not iid with clustered data...

"Unconditional Means" Model

- Basic MLM with Random Intercept (plus One-Way ANOVA with Random Effects)
- Useful as a baseline...

Level-1 equation (students): \( \text{MATHACH}_i = \beta_0 + \epsilon_i \) where \( \epsilon_i \sim N(0, \sigma^2) \)

Level-2 equation (schools): \( \beta_0 = \gamma_0 + \mu_0 \) where \( \mu_0 \sim N(0, \tau_0) \)

- \( j \) indexes schools; \( i \) indexes students
- \( \text{MATHACH}_i \) = Student Math Achievement (the DV)
- \( \beta_0 \) = Intercepts for the student’s school (the value for each students’ school)
- \( \epsilon_i \) = Student-level residual with a variance of \( \sigma^2 \) (random effect)
- \( \gamma_0 \) = Overall mean for schools (fixed effect)
- \( \mu_0 \) = Deviation from mean for each school
- \( \epsilon_i \) = School-level residual with a variance of \( \tau_0 \) (fixed effect)
- \( \tau_0 \) measures how much students in the same class vary together.

And by substitution:

\( \text{MATHACH}_i = \gamma_0 + \mu_0 + \epsilon_i \)

"Unconditional Means" Model (part 2)

And by substitution:

\( \text{MATHACH}_i = (\gamma_0 + \mu_0) + \epsilon_i \)
"Unconditional Means" Model in SAS

\[ \text{MATHACH}_i = (\mu + \alpha_i) + \epsilon_i \]

- proc mixed noclprint covtest;
- class school;
- model MATHACH = /solution;
- random intercept /subject=school;

From Singer (1998) results on pg. 329 (code on pg. 327)

"Unconditional Means" Model in SAS (part 2)

- proc mixed noclprint covtest;
- class school;
- model MATHACH = /solution;
- random intercept /subject=school;

- Noclprint – prevents output of information of classification variables (listed in class statement)
- Covtest – output hypothesis tests for random effects
- Class Statement – declare variable to be a classification variable
- Model Statement – list fixed effects here (intercept is included by default)
- Solution – after "/" in model statement; print solutions for fixed effects
- Random Statement – declare random effects (level-1 error/random effect included by default)
- Subject – specify the level-2 unit

Intraclass Correlation

- \( \beta \) measures the amount of clustering of level-1 observations within level-2 units.
- If this is very small, you may not need a MLM.
  - You will likely have few significant level-2 predictors.
  - There is little dependence due to clustering in level-2 units.

\[ \beta = \frac{\sigma^2_\text{e}}{\sigma^2_\text{e} + \sigma^2_\text{u}} \]

\( p = .18 \) in this example
“Including Effects of School Level Predictors”

- Just add a level-2 covariate to the “Unconditional Means” model.
  - Notice that the covariate is in the intercept equation since it is at the school level (level-2). It is Grand-mean centered.
  - Additional level-2 variables can be added in the same way.

Level 1: \( \text{MATHCIR} = \beta_0 + r_j \)

Level 2: \( \beta_0 = \gamma_{00} + \gamma_{01} \times \text{MEANSES} + u_{0j} \)

where \( u_{0j} \sim N(0, \sigma^2) \)

By substitution:

\( \text{MATHCIR}_j = (\gamma_{00} + \gamma_{01} \times \text{MEANSES}_j + u_{0j}) + r_j \)

Centering

- Reminder: The intercept is the value of the DV when all covariates are equal to 0.
- Centering ensures that the intercept will have a meaningful interpretation (it will also affect the significance of the term).
- Helpful with cross-level interactions, since the intercept can end representing an extreme value.
- Do not center your DV in any way.
- It is important to tell your audience how you centered your variables for MLMs—this changes the interpretation of the results!

Centering (types)

- Grand-mean centering
  - Difference with overall mean e.g. \( \text{SES}_j = \overline{\text{SES}}_j \) or just \( \overline{\text{SES}} \)
  - Advised for level-2 variables. Can be helpful in OLS as well...
  - Might seem strange for dichotomous variables...
  - Changes interpretation of intercept. If all variables are centered, the intercept represents the value for the individual with average values for all the variables.
- Group-mean centering
  - Difference with group mean e.g. \( \text{SES}_j = \overline{\text{SES}}_j \)
  - Almost always needed for level-1 variables in MLMs.
  - Isolates the level-1 effect.
  - Will change the interpretation of slopes and intercepts.
“Including Effects of School Level Predictors” in SAS

\[ MATHACH_t = (y_{1s} + y_{1s} + MEANSES_t + u_{1s}) + \epsilon_{t} \]

```
proc mixed noclprint covtest;
  class school;
  model MATHACH = MEANSES/solution ddfm=bw;
  random intercept /subject=school;
```

```
Singer Pg. 331
```

```
Explained Variance

- The change in the size of random effects can be computed between models (usually using the Unconditional Means Model as the baseline) as a measure of explained variance.
  - In this instance the change in the level-2 variance was (8.61 - 2.65)/8.61 ≈ 0.69.
  - MEANSES explained 69% of the total variance between schools.
- There was little to no change in the level-1 variance, but it can be computed and interpreted in the same way.

- Warning: variances can change unexpectedly (although usually just small changes) when additional random effects and covariates are included.

```
Singer Pg. 331
```
“Including Effects of Student-Level Predictors”

- Add a level-1 covariate and random effect to the Unconditional Means Model.
  - Hypothesis: 1. SES affects MATHACH (fixed effect) 2. The effect of SES on MATHACH varies across schools.

\[
M_{\text{MATHACH}} = \beta_0 + \beta_1(\text{SES}_j - \bar{\text{SES}}) + \epsilon_j
\]

- In this model they are allowed to covary.

\[
\begin{align*}
\beta_0 &= \gamma_{00} + u_{0j} \\
\beta_1 &= \gamma_{10} + u_{1j} \\
\epsilon_j &= \sigma_{e}^2 \begin{bmatrix} e_{0j}^2 & e_{1j}^2 & e_{2j}^2 \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\gamma_{00} &\sim N(0, \sigma_0^2) \\
\gamma_{10} &\sim N(0, \sigma_1^2) \\
\sigma_e^2 &\sim (0, \tau_{e}^2)
\end{align*}
\]

\[
\begin{align*}
\text{tau}_{0} &\text{ The variance in the intercept} \\
\text{tau}_1 &\text{ The variance in the slope of SES} \\
\text{tau}_{01} &\text{ The covariance between the intercepts and slopes. Here tau}_{01} = \tau_1 \text{. In this model they are allow to covary.}
\end{align*}
\]

“Including Effects of Student-Level Predictors” (part 2)

\[
M_{\text{MATHACH}} = \beta_0 + \beta_1(\text{SES}_j - \bar{\text{SES}}) + \epsilon_j
\]

By substitution:

\[
M_{\text{MATHACH}} = (\gamma_{00} + u_{0j}) + (\gamma_{10} + u_{1j})(\text{SES}_j - \bar{\text{SES}}) + \epsilon_j
\]

\[
M_{\text{MATHACH}} = (\gamma_{00} + u_{0j}) + u_{1j}(\text{SES}_j - \bar{\text{SES}}) + \epsilon_j
\]

“Including Effects of Student-Level Predictors” in SAS

```sas
proc mixed; class school; model MATHACH = CSSES solution ddfm=lsmeans; random intercept CSSES /subject=school type=un;
```

Singer pg 335 (334)
“Including Effects of Student-Level Predictors” in SAS (part 2)

proc mixed noclprint covtest noitprint;
  class school;
  model MATHACH = CSSES /solution ddfm=bw notest;
  random intercept CSSES /subject=school type=un;

Notprint – suppress output on iterations
Notest – suppresses some tests for fixed effects that are not needed
Type=un – specifies that the random be allows the random effects to covary

---

“Including Both Level-1 and Level-2 Predictors”

- Add in the level-2 variables to the previous model and include cross-level interactions.
  - Tip: slowly build your models and watch for major shifts in coefficients. This may indicate a problem.

MATHACH = β0 + μ0 + (μ0) + c0 + Ω0
β1 = τ0 + (τ0) + Ω0 + c1
β2 = τ1 + (τ1) + Ω1 + c2

By substitution:

MATHACH = β0 + μ0 + (μ0) + c0 + Ω0 + (τ0) + (τ0) + Ω0 + c1 + (τ1) + (τ1) + Ω1 + c2

Singer pg. 335 – 336

“Including Both Level-1 and Level-2 Predictors” in SAS

proc mixed noclprint covtest noitprint;
  class school;
  model MATHACH = MEANSES SECTOR CSES MEANSES*CSES SECTOR*CSES /solution ddfm=bw notest;
  random intercept CSSES /subject=school type=un;

Singer pg. 337 – 338
“Including Both Level-1 and Level-2 Predictors” in SAS

- Do we need the random effect for CSES? It is not significant... Drop it from the model...

\[
\begin{align*}
\mathit{MATHACH}_{ij} &= \beta_0 + \beta_1(\mathit{MEANSES} - \mathit{MEANS}) + e_{ij} \\
\beta_1 &= \mu_1 + \gamma_1\mathit{MEANSES} + \gamma_2\mathit{SECTOR} + u_{1ij} \\
\beta_2 &= \mu_2 + \gamma_2\mathit{MEANSES} + \gamma_3\mathit{SECTOR} + u_{2ij}
\end{align*}
\]

where \( e_{ij} \sim N(0, \sigma^2) \)

\[
\begin{align*}
\mathit{MATHACH}_{ij} &= \beta_0 + \beta_1(\mathit{MEANSES} - \mathit{MEANS}) + u_{ij} \\
\beta_1 &= \mu_1 + \gamma_1\mathit{MEANSES} + \gamma_2\mathit{SECTOR} + u_{1ij} \\
\beta_2 &= \mu_2 + \gamma_2\mathit{MEANSES} + \gamma_3\mathit{SECTOR} + u_{2ij}
\end{align*}
\]

Model Building

- Build your models slowly and watch for major shifts in coefficients. This may indicate a problem.
  - MLMs sometimes have "squishy variance," particularly when models are complicated. This can make measuring variance in accurate between models.
  - Significance tests of variance components make a number of assumptions, such as normality, which is often not the case.
  - Differences between models are tested with the \(-2\) Log Likelihood statistic and other Log Likelihood based statistics. Remember to use ML...
  - AIC, AICC, BIC incorporate penalties into \(-2LL\) based on the number of covariates and, in some cases (just AIC and BIC, I think), the number of cases (N).
  - Significance tests using \(-2LL\) follow the chi-square distribution. Models must be nested.
  - Other statistics also have rules of thumb for determining "significance". Models need not be nested.
  - Difference in AIC < 2 means they are very similar; >10 lots of evidence that they are different.

Comparison of Models with and without a Random Slope

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<th>With Random Slope</th>
<th>Without Random Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2) Res Log Likelihood</td>
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<td>46504.8</td>
</tr>
<tr>
<td>AIC (Smaller is Better)</td>
<td>46511.7</td>
<td>46508.8</td>
</tr>
<tr>
<td>AICC (Smaller is Better)</td>
<td>46511.7</td>
<td>46508.8</td>
</tr>
<tr>
<td>BIC (Smaller is Better)</td>
<td>46524.0</td>
<td>46514.9</td>
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</table>

Using ML:

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<th>With Random Slope</th>
<th>Without Random Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2) Log Likelihood</td>
<td>46496.4</td>
<td>46497.4</td>
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<tr>
<td>AIC (Smaller is Better)</td>
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<td>46517.4</td>
</tr>
<tr>
<td>AICC (Smaller is Better)</td>
<td>46516.4</td>
<td>46517.4</td>
</tr>
<tr>
<td>BIC (Smaller is Better)</td>
<td>46547.2</td>
<td>46538.0</td>
</tr>
</tbody>
</table>
What are Individual Growth Models?

- Method used to analyze change over time... “for data with repeated measures.”
  - Treats time more flexibly than other methods.
  - Can allow the effect of time to randomly vary
  - Can predict change over time (explain the random effect for time).
  - Can also predicting differences in the average outcomes (between individuals).
  - Do not need equally spaced measures (in time).
  - Can handle missing data.
  - But... can get very complicated and difficult to interpret.
- Remember: the structure of your data is very important, it needs to be in “long” or “person-period” format.

“An Unconditional Linear Growth Model”

- Very similar to the previous models we have run. Notion is just a bit different.
  - This is usually not the place to start with these types of models, but it serves as a good transition... 

\[ Y_{ij} = \beta_0 + \beta_1 \times \text{TIME}_{ij} + \epsilon_{ij} \] where \( \epsilon_{ij} \sim N(0, \sigma^2) \)

\[ \mu_{ij} = \mu_0 + \mu_1 \text{ where } \mu_0 \sim N(0, \tau_{00}) \]

\[ \sigma_{ij} = \sigma_0 + \sigma_1 \text{ where } \sigma_1 \sim N(0, \tau_{11}) \]

\[ \text{Cov}(\epsilon_{ij}) = \begin{pmatrix} 0 & \tau_{01} \\ \tau_{10} & \tau_{11} \end{pmatrix} \]

By substitution:

\[ Y_{ij} = (\beta_0 + u_{i0}) + (\beta_1 + u_{i1}) \times \text{TIME}_{ij} + \epsilon_{ij} \]

\[ Y_{ij} = (\beta_0 + u_{i0}) + \beta_1 \times \text{TIME}_{ij} + u_{i1} \times \text{TIME}_{ij} + \epsilon_{ij} \]
“An Unconditional Linear Growth Model” in SAS

\[ Y_{ij} = (\beta_0 + \theta_1 \text{TIME}_{ij}) + u_{i0} + u_{i1} \text{TIME}_{ij} + \epsilon_{ij} \]

\[ \begin{bmatrix} Y_{ij} \\ \theta_1 \\ u_{i0} \\ u_{i1} \\ \epsilon_{ij} \end{bmatrix} = \begin{bmatrix} 1 \\ \text{TIME}_{ij} \\ 1 \\ 1 \\ 1 \end{bmatrix} \beta + \begin{bmatrix} u_{i0} \\ u_{i1} \end{bmatrix} + \epsilon_{ij} \]

proc mixed noclprint covtest;
  class id;
  model y = time / solution ddfm=bw notest;
  random intercept time / subject=id type=un;

Singer pg. 342 (341)

Data is from:
John B. Willett
Review of Research in Education

“An Unconditional Linear Growth Model” in SAS (part 2)

- These is really nothing here that we haven't already seen—just different variables.

proc mixed noclprint covtest;
  class id;
  model y = time / solution ddfm=bw notest;
  random intercept time / subject=id type=un;

id = person id
y = A simulated proficiency scores for an “opposite-naming task.”
Time = It is coded from 0-3.
Level-2 effects are interpreted as changes between individuals where time=0. Here, 0 corresponds to their initial status.

“Linear Growth Model with a Person-Level Covariate”

- Add in a covariate to the previous model with a cross-level interaction.

\[ Y_{ij} = \beta_0 + \beta_1 \text{TIME}_{ij} + \theta_0 + \theta_1 \text{covariate}_{ij} + \epsilon_{ij} \]

\[ \begin{bmatrix} Y_{ij} \\ \theta_0 \\ \theta_1 \end{bmatrix} = \begin{bmatrix} 1 \\ \text{TIME}_{ij} \\ \text{covariate}_{ij} \end{bmatrix} \beta + \begin{bmatrix} \theta_0 \\ \theta_1 \end{bmatrix} + \epsilon_{ij} \]

By substitution:
\[ Y_{ij} = (\beta_0 + \beta_1 \text{TIME}_{ij}) + u_{i0} + u_{i1} \text{TIME}_{ij} + \epsilon_{ij} \]

\[ Y_{ij} = (\beta_0 + \beta_1 \text{TIME}_{ij}) + (\beta_0 + \beta_1 \text{TIME}_{ij}) + (\beta_0 + \beta_1 \text{TIME}_{ij}) + \epsilon_{ij} \]
“A Linear Growth Model with a Person-Level Covariate” in SAS

\[
Y_j = (\beta_0 + \beta_1 \text{TIME}_j + u_0) + \beta_2 \text{TIME}_j + \beta_3 \text{TIME}_j \text{COVAR} + \epsilon_j
\]

\[\text{proc mixed noclprint covtest;}\]
\[\text{class id;}\]
\[\text{model y = time ccovar time*ccovar / solution ddfm=bb notest;}\]
\[\text{random intercept time / subject=id type=un gcorr;}\]

- Gcorr - prints out correlations between random effects.


“Exploring the Structure of Variance Covariance Matrix Within Persons”

- Now to take a step back and model the within-person error. This error likely to be more complex than can be handled with the previous models.
  - Below, \(\epsilon\) corresponds to an error-covariance matrix that is selected by the researcher.

\[Y_j = \mu_0 + \mu_1 (\text{TIME}_j) + \epsilon_j \text{ where } \epsilon_j \sim N(0,\Sigma)\]

- After substitution:

\[Y_j = \mu_0 + \mu_1 (\text{TIME}_j) + \epsilon_j\]


Error-Covariance Matrices

- “Variance Components” is the most basic configuration. It assumes no covariation between units (usually time).
- A common choice is compound symmetry:

\[
\begin{bmatrix}
\sigma^2 & \rho & \rho & \rho \\
\rho & \sigma^2 & \rho & \rho \\
\rho & \rho & \sigma^2 & \rho \\
\rho & \rho & \rho & \sigma^2
\end{bmatrix}
\]

- Another common structure is 1st order autoregressive:

\[
\begin{bmatrix}
1 & \rho & \rho & \rho \\
\rho & 1 & \rho & \rho \\
\rho & \rho & 1 & \rho \\
\rho & \rho & \rho & 1
\end{bmatrix}
\]

- You should try many different structures and compare -2LL, AIC, BIC scores to select the best structure.
“Exploring the Structure of Variance Covariance Matrix Within Persons” in SAS

\[ T_y = \beta_0 + \beta_1 (\text{TIME}_i) + \epsilon \]
where \(\epsilon \sim N(0, \sigma^2)\) and \(\epsilon\) is compound symmetric.

```
proc mixed noclprint covtest noitprint;
  class id wave;
  model y = time / solution notest;
  repeated wave / subject=id type=cs r;
```

“Wave” is the same as “time”. One version for the class statement and one for the model statement. The Repeated Statement – specifies structure for the error term - the rows and columns of the error covariance matrix. Subject=id – the error term is within individuals (waves are the level-1 unit). r – prints the matrix for the error term (the “R” matrix).

Intercepts and Slopes as Outcomes

- Adding in a covariate, random intercept, and random slope for time.

\[
\begin{align*}
  y_j &= \beta_0 + \beta_1 (\text{TIME}_i) + \epsilon_{ij} \\
  x_{1j} &= \beta_0 + \beta_1 (\text{COVAR}) + \epsilon_j \\
  x_{2j} &= \beta_0 + \beta_1 (\text{COVAR}) + \epsilon_j
\end{align*}
\]
where \(\epsilon_{ij} \sim N(0, \sigma^2)\) and \(\epsilon_j \sim N(0, \tau_{11})\).

By substitution:

\[
\begin{align*}
  y_j &= (\beta_0 + \epsilon_j) + (\beta_1 (\text{TIME}_i) + \beta_1 (\text{COVAR}) + \epsilon_j)
\end{align*}
\]

Intercepts and Slopes as Outcomes in SAS

```
proc mixed noclprint covtest notest;
  class id wave;
  model y = time time*ccovar / solution ddfm=bw notest;
  random intercept time / subject=id type=un g;
  repeated wave / type=ar(1) subject=id r;
```

Singer pg. 346
Conclusion

- MLMs are flexible and have numerous applications, but are complex tools that need to be handled with care.
- Along with the classic two- and even three-level models, MLMs can be used to conduct longitudinal analyses.
- This workshop only covered the basics—additional attention needs to be given to residual diagnostics and other issues before we can be confident about inferences from MLMs.